H-2324

M. A. (Part - I) (Mathematics) (External) Examination

May/June - 2018

Paper - 405 : Graph Theory & Discrete Structure

Time : 3 Hours] [Total Marks : 100

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ M. A. (PART - 1) (MATHEMATICS) (EXTERNAL)

Name of the Subject :
☛ P. - 405 : GRAPH THEORY & DISCRETE STRUCTURE

☛ Subject Code No. : 2 3 2 4 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figures to the right indicate marks.

(4) Follow the usual notations and conventions.

1 Attempt any FIVE. 20

1. If the intersection of two paths is a disconnected graph then prove that the union of those two paths has at least one circuit.
2. Prove that a simple graph with n vertices and k components can have at most (n- k)(n-k+1)/2 edges.
3. Prove that an infinite graph with finite number of edges must have infinite number of isolated vertices.
4. Prove that a connected graph G remains connected after removing an edge e_i from G iff e_i is in some circuit in G.
5. Prove that every finite lattice is complete.
6. Discuss clock algebra.
7. Prove that 1 is the only complement of 0 in a lattice.

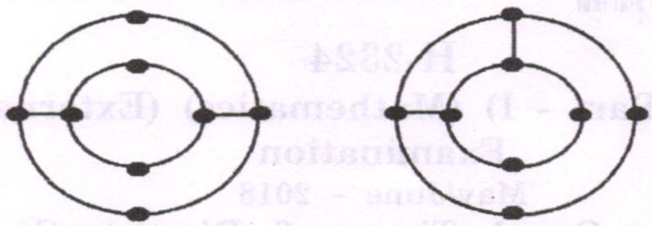
2 (a) Prove that a given connected graph G is an Euler graph iff all vertices of G are of even degree. 7

(b) Consider the utilities problem for four houses and four utilities. Explain and discuss its solution. 7

(c) Show that the distance between vertices of a connected graph is a metric. 6

OR

2 **(a)** Are the two graphs isomorphic? Why? 7



(b) State and prove Euler's formula. 7

(c) Prove that any connected graph with n vertices and n-1 edges is a tree. 6

3 **(a)** Prove that a planar graph may be embedded in a plane such that any specified region can be made the infinite region. 7

(b) Prove that an Euler graph G is arbitrarily traceable from vertex v in G iff every circuit in G contains v. 7

(c) Prove that in a connected graph G, a vertex v is a cut-vertex iff there exist at least two edges x and y incident on v such that no circuit in G includes both x and y. 6

OR

3 **(a)** In a given connected weighted graph G, suppose there exists an edge e_s whose weight is smaller than that of any other in G. prove that every shortest spanning tree in G must contains e_s. 7

(b) What is the maximum possible height of an n-vertex binary tree? 7

(c) Prove that Kuratowski's second graph is non-planar. 6

4 **(a)** Let $S = \{a, b, c\}$ then prove that $< \rho(S), \cup >$ and $< \rho(S), \cap >$ are monoids. 7

(b) Prove that the direct product of two commutative semigroups is a commutative semigroup. 7

(c) For the set of natural numbers N, prove that $< N, + >$ is a semigroup. Is the set odd non-negative integers form a subsemigroup for $< N, + >$? Justify your answer. 6

OR

4 **(a)** Prove that, no element of V^* is invertible, except the empty string. 7

(b) Prove that $< Z_m, +_m >$ and $< Z_m^*, \oplus_m >$ are isomorphic. 7

(c) State and prove the Fermat's theorem. 6

5 **(a)** State and prove the distributive inequality for a lattice. 7

(b) Any antiatom which is $\geq a$ must appear in the meet representation of the element a. 7

(c) Prove that every chain is a distributive lattice. 6

OR

5 **(a)** Show that in a complemented distributive lattice, 7

$$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$$

(b) Prove that for any mapping from a Boolean algebra which preserves the operation $*$ and $\oplus$, the image set is also a Boolean algebra. 7

(c) Explain the Quine Mc-clusky algorithm to minimize a Boolean function. 6
